MASS TRANSFER IN A CYLINDRICAL CHANNEL

WITH EVAPORATION AND CONDENSATION AT THE WALLS

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A model kinetic equation is used to investigate the transfer of material evaporated from the walls of a channel which has a temperature gradient along its axis.

Relatively few papers have used a kinetic equation to study mass transfer in channels with evaporation and condensation. Mass transfer in a cylindrical channel of finite length is treated in [1, 2] for a broad range of Knudsen numbers. The temperature of the channel walls is assumed constant, and the condensation coefficient is taken as unity. Evaporation occurs only from the bottom of the channel in [1], but from both the bottom and sides in [2].

We investigate the transfer of wall material in a long cylindrical channel of radius R which has a temperature gradient along the channel axis z. We describe the state of the vapor by the Boltzmann equation with a Bhatnagar–Gross–Krook model of the collision integral. The boundary conditions are formulated under the assumption that both the reflected molecules and those evaporated from the walls have a Maxwellian distribution of velocities characterized by the wall temperature. The condensation coefficient β is assumed temperature independent, i.e., constant along the channel length. Under these assumptions the boundary condition for the distribution function can be written in the form

$$f_R(\mathbf{nv}>0)=\left[\rho+\frac{1-\beta}{\beta}\left(\frac{2\pi m}{kT}\right)^{1/2}q_R\right]\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT}\right), \tag{1}$$

where $T=T(z)$ is the given temperature distribution of the channel walls in °K, $\rho=\rho(z)$ is the saturated vapor density at the temperature $T(z)$, and $q_R(z)$ is the radial component of the mass flux density at the channel wall.

For small temperature gradients the distribution function of the vapor in a given cross section of the channel is negligibly different from that of particles moving from the channel wall (1), and therefore we seek it in the form

$$f=f_R[1+h(\mathbf{r},\ z,\ \mathbf{v})]; \tag{2}$$

$h \ll 1$ and at the channel wall satisfies the boundary condition

$$h_R(\mathbf{nv}>0)=0. \tag{3}$$

The linearized Boltzmann equation for the problem under consideration can be written in the form

$$\begin{aligned}\mathbf{c}_\perp\frac{\partial h}{\partial \mathbf{r}}&+c_z\lambda\left[\frac{1}{\rho}\frac{d\rho}{dz}+\left(c^2-\frac{3}{2}\right)\frac{1}{T}\cdot\frac{dT}{dz}+\right.\\&\left.+\frac{1-\beta}{\beta}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{1}{\rho}\cdot\frac{dq_R}{dz}+\frac{\partial h}{\partial z}\right]=\left(\frac{\rho_V}{\rho}-1\right)+\\&+\left(c^2-\frac{3}{2}\right)\frac{\rho_V}{\rho}\cdot\frac{T_V-T}{T}+2\left(\frac{m}{kT}\right)^{1/2}\frac{\mathbf{cq}}{\rho}+\\&+\frac{1-\beta}{\beta}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{q_R}{\rho}-h,\end{aligned} \tag{4}$$

where $\lambda=(\mu/\rho)(2m/kT)^{1/2}$ is the mean free path; μ is the viscosity of the vapor; $\mathbf{r}$ is the two-dimensional radius-vector in a plane perpendicular to the z axis in units of λ; and $\mathbf{c}$ is the dimensionless molecular velocity.

The term $\partial h/\partial z$ is ordinarily omitted in solving problems of the flow of a gas in a cylindrical channel [1, 4, 5]. In the present case this approximation implies that the essential dependence of the distributed function on z is expressed in terms of ρ, T, and q_R. Integrating Eq. (4) along the characteristics, we obtain

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$$h = \int_0^{v} \frac{1}{c_\perp} \exp\left(-\frac{s}{c_\perp}\right)\left[-\lambda c_z\left(\frac{1}{\rho}\frac{d\rho}{dz} + \left(c^2 - \frac{3}{2}\right)\frac{1}{T}\frac{dT}{dz} + \right.\right.$$

$$\left. + \frac{1-\beta}{\beta}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{1}{\rho}\frac{dq_R}{dz}\right) + \left(\frac{\rho_V}{\rho} - 1\right) +$$

$$\left. + \left(c^2 - \frac{3}{2}\right)\frac{\rho_V}{\rho}\cdot\frac{T_V - T}{T} + 2\left(\frac{m}{kT}\right)^{1/2}\frac{\mathbf{cq}}{\rho} + \frac{1-\beta}{\beta}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{q_R}{\rho}\right] ds. \tag{5}$$

Multiplying (5) by $(1/\pi^{3/2})\,\rho(2kT/m)^{1/2}\,c_z\exp(-c^2)$ and integrating with respect to c, we obtain an integral equation for the mass flux density along the z axis:

$$q_z - \frac{1}{\pi}\int \frac{T_0(|\mathbf{r}-\mathbf{r}_1|)}{|\mathbf{r}-\mathbf{r}_1|} q_z(\mathbf{r}_1)\, d\mathbf{r}_1 = \frac{\lambda}{2\pi}\left(\frac{2kT}{m}\right)^{1/2}\left[-\left(\frac{d\rho}{dz} + \right.\right.$$

$$\left.\left. + \frac{1-\beta}{\beta}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{dq_R}{dz}\right)\int \frac{T_0(|\mathbf{r}-\mathbf{r}_1|)}{|\mathbf{r}-\mathbf{r}_1|}d\mathbf{r}_1 - \frac{\rho}{T}\frac{dT}{dz}\int\frac{T_2(|\mathbf{r}-\mathbf{r}_1|)}{|\mathbf{r}-\mathbf{r}_1|}d\mathbf{r}_1\right], \tag{6}$$

where $T_n(x) = \int_0^\infty t^n \exp\left(-t^2 - \frac{x}{t}\right) dt$ (see [6]).

Equation (6) is solved by the Bubnov–Galerkin method, using for q_z the approximation

$$q_z = A(z)\, r^2 + B(z).$$

It is shown in [5] that when this approximation is used in the integral equation which describes Poiseuille flow and differs from (6) only on the right-hand side, the result is in good agreement with the numerical solution of this equation given in [4]. The two solutions do not differ by more than 1% for Knudsen number from 0.1 to ∞. Performing the calculation, we obtain the following expression for the mass flux along the channel axis:

$$G = -\pi R^3\left(\frac{m}{2kT}\right)^{1/2}\left[Q_1\left(\frac{dP}{dz} + \frac{1-\beta}{\beta}\left(\frac{2\pi kT}{m}\right)^{1/2}\frac{dq_R}{dz}\right) + Q_2\frac{P}{T}\frac{dT}{dz}\right], \tag{7}$$

where P is the pressure of the saturated vapor at temperature T(z). The coefficient Q_1 and Q_2 correspond to those in [7]. In the description of nonisothermal Poiseuille flow in [7], Q_1 represents the dimensionless flux resulting from the pressure gradient and Q_2 the flux resulting from the temperature gradient.

For $\beta = 1$, Eq. (7) takes the form

$$G_1 = -\pi R^3\left(\frac{m}{2kT}\right)^{1/2}\left[Q_1\frac{dP}{dz} + Q_2\frac{P}{T}\frac{dT}{dz}\right]. \tag{8}$$

Equation (8) determines mass transfer over the whole range of Knudsen numbers and has the same form as the expression describing nonisothermal Poiseuille flow in a cylindrical channel. We note, however, that in this case Eq. (8) contains the saturated vapor pressure gradient determined by the given temperature distribution of the channel walls.

In the limiting case of large Knudsen numbers (8) becomes

$$G_1 = -\frac{8}{3}R^3\sqrt{\frac{\pi m}{2k}}\,\frac{d}{dz}\frac{P}{\sqrt{T}}. \tag{9}$$

At the other extreme, when Kn≪ 1 we have

$$G_1 = -\frac{\pi R^4}{8\mu}\rho\frac{dP}{dz}. \tag{10}$$

For $\beta \neq 1$, Eq. (7) involves the flux at the channel wall. Since this flux is related to the axial flux by the equation of continuity

$$q_R = -\frac{1}{2\pi R}\cdot\frac{dG}{dz}, \tag{11}$$

we obtain a second-order differential equation for G:

$$\frac{d^2G}{dz^2} - \alpha^2 G = -\alpha^2 G_1, \tag{12}$$

where $\alpha^2 = (2/\sqrt{\pi}) \cdot [\beta/(1-\beta)] \cdot (1/Q_1) \cdot (1/R^2)$. The quantities Q_1 and α are functions of the Knudsen number; i.e., they generally depend on z. For large and intermediate Knudsen numbers, however, this dependence is very weak [8]. For $Kn \ll 1$, $Q_1 = {}^1/_4 Kn^{-1}$. Even in this case, however, the dependence of Q_1 on z can be neglected for small temperature and pressure drops along the channel, and the solution of Eq. (12) can be written in the form

$$G = C_1 \exp(\alpha z) + C_2 \exp(-\alpha z) + \\ + \frac{\alpha}{2} \exp(-\alpha z) \int_{z_1}^{z} G_1(\xi) \exp(\alpha\xi)\, d\xi - \frac{\alpha}{2} \exp(\alpha z) \int_{z_1}^{z} G_1(\xi) \exp(-\alpha\xi)\, d\xi. \quad (13)$$

Determining C_1 and C_2 from the condition that the flux must be bounded as $z \to \pm\infty$, we obtain

$$G = \frac{\alpha}{2} \exp(-\alpha z) \int_{-\infty}^{z} G_1(\xi) \exp(\alpha\xi)\, d\xi + \frac{\alpha}{2} \exp(\alpha z) \int_{z}^{\infty} G_1(\xi) \exp(-\alpha\xi)\, d\xi. \quad (14)$$

Thus, the mass flux for an arbitrary value of the condensation coefficient can be expressed in terms of the mass flux for $\beta = 1$.

Integrating by parts successively, we can write (14) in the form

$$G = \sum_{i=0}^{\infty} \frac{1}{\alpha^{2i}} \cdot \frac{d^{2i} G_1}{dz^{2i}}. \quad (15)$$

The first term of series (15) represents the mass flux for $\beta = 1$ and does not depend on the condensation coefficient. Suppose $L \gg R$ is a characteristic length over which there is an appreciable change in the temperature and its derivatives. Then when the condition

$$(R/L)^2 Q_1 \ll \beta/(1-\beta) \quad (16)$$

is satisfied the remaining terms of the series do not contribute appreciably to the flux; i.e., mass transfer is independent of the condensation coefficient. For large and intermediate Knudsen numbers Q_1 is close to unity and condition (16) can be satisfied over a broad range of β values. For $Kn \ll 1$, Eq. (16) is considerably more stringent:

$$\left(\frac{R}{L}\right)^2 \ll 4\mathrm{Kn} \frac{\beta}{1-\beta}.$$

It is known that the processes of evaporation and condensation can be considered as a special case of a reversible heterogeneous first-order reaction. Condition (16) implies that the reaction proceeds in the diffusion regime when the effective reaction rate is determined by the yield and removal of the reacting materials and not by the reaction rate at the wall. This also explains why mass transfer is independent of the condensation coefficient when (16) is satisfied.

NOTATION

f, distribution function; v, molecular velocity; $v_\perp$, component of molecular velocity in a plane perpendicular to the channel axis; **n**, unit vector normal to the channel surface; φ, angle between **n** and $v_\perp$; m, molecular weight; k, Boltzmann constant; ρ_V, vapor density in channel; T_V, vapor temperature in channel in °K; λ, mean free path of vapor molecules; δ, radius of channel in units of λ; $b = -r\cos\varphi + \sqrt{\delta^2 - r^2 \sin^2\varphi}$, characteristic length from point r to channel wall; Kn, Knudsen number. Indices: R, value of quantity at channel wall.

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